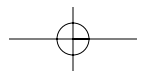
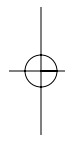
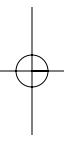


Probability in Petroleum and Environmental Engineering



Probability in Petroleum and Environmental Engineering

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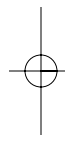
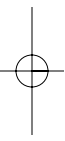
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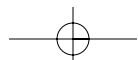
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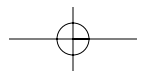
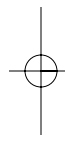
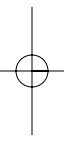
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*This book is dedicated to
His Majesty King Bhumibol Adulyadej
and Her Majesty Queen Sirikit of Thailand
for their relentless efforts
to raise the standard of living
of their wonderful people.*



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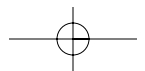
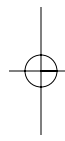
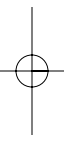
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PREFACE

From thermodynamics to quantum mechanics to modern communication technology, important concepts from probability and statistics have become increasingly dominant in many areas of science and engineering over the past 150 years. This book plays a vital role of introducing the fundamental ideas of probability and statistics in a way that is directly applicable to practical situations in environmental and petroleum engineering, and also is sound mathematically. For example, estimation of pollution levels in air, water, and soil, as well as the characterization of petroleum reservoirs, are appropriately illustrated in a probabilistic context. Statistical testing and measures of significance are explained clearly, and each chapter concludes with an excellent set of questions and exercises.

Solomon W. Golomb, Academician,
University Professor of Engineering and Mathematics,
University of Southern California, Los Angeles



LIST OF NOTATIONS

<i>Symbol</i>	<i>Meaning</i>
$A \Rightarrow B$	Statement B follows from statement A ; A implies B .
$A \Leftrightarrow B$	Statements A and B are equivalent.
❖	Beginning and end of proof.
$\forall x$	For all x .
$\exists x$	x exists.
$x \in X$	Element x belongs to set X .
$f: X \rightarrow Y$	f is a function with domain X and range Y . Customary notation is $y = f(x)$, $x \in X$, $y \in Y$.
$x \in A$	x is an element of set A .
$x \notin A$	x is not an element of set A .
$A \cup B$	Denotes union of two sets A and B .
$A \cap B$	Denotes intersection of two sets A and B .
$A \subset B$	A is a subset of B .
$A = B$	A and B are equal if $A \subset B$ and $B \subset A$.
\emptyset	Denotes empty set.

